Name:_____________________________________ Date:____________________________

Pre-Calculus 11: HW 1.2 Arithmetic Series Solution

1. Find the sum of the following arithmetic series. Use the formula: $S_n = \frac{n}{2} \times (a + t_n)$

i) $13+17+21+25+.....+t_8$ $a=13, d=4, n=8$ $t_8 = 13+(8-1)4$ $t_8 = 13+28$ $t_8 = 41$ $S_8 = \frac{8}{2}(13+41)$ $S_8 = 4(54)$ $S_8 = 216$	ii) $-5+1+7+13+.....+t_{12}$ $a=-5, d=6, n=12$ $t_{12} = -5+(12-1)6$ $t_{12} = -5+66$ $t_{12} = 61$ $S_{12} = \frac{12}{2}(-5+61)$ $S_{12} = 6(56)$ $S_{12} = 336$
iii) $9+14+19+24+.....+104$ $a=9, d=5, t_n=104$ $104 = 9+(n-1)5$ $104 = 9+5n-5$ $100 = 5n$ $20 = n$ $S_{20} = \frac{20}{2}(9+104)$ $S_{20} = 10(113)$ $S_{20} = 1130$	iv) $11+28+45+62+.....+368$ $a=11, d=17, t_n=368$ $368 = 11+(n-1)17$ $368 = 11+17n-17$ $374 = 17n$ $22 = n$ $S_{22} = \frac{22}{2}(11+368)$ $S_{22} = 11(379)$ $S_{22} = 4169$
v) $\frac{24}{3} + \frac{14}{3} + \frac{4}{3} + \frac{-6}{3} + ... \; t_{10}$ $a=8, d=-10/3, n=10$ $t_{10} = 8+(10-1)\,{-10/3}$ $t_{10} = 8-30$ $t_{10} = -22$ $S_{10} = \frac{10}{2}(8-22)$ $S_{10} = 5(-14)$ $S_{10} = -70$	vi) $6 + \frac{9}{2} + 3 + \frac{3}{2} + ... \; -28.5$ $a=6, d=-1.5, t_n=-28.5$ $-28.5 = 6+(n-1)(-1.5)$ $-28.5 = 7.5-1.5n$ $-36 = -1.5n$ $24 = n$ $S_{24} = \frac{24}{2}(6-28.5)$ $S_{24} = 12(-22.5)$ $S_{24} = -270$

2. Given each series and the information provided, find the missing value:

a) $a=12, \;\; n=20, \;\; d=4, \;\; S_{20}=?$ $a=12, d=4, n=20$ $t_{20} = 12+(20-1)4$ $t_{20} = 12+76$ $t_{20} = 88$ $S_{20} = \frac{20}{2}(12+88)$ $S_{20} = 10(100)$ $S_{20} = 1000$	b) $a=7, \;\; t_n=42, \;\; S_n=857.5, \;\; n=?$ $a=7, t_n=42, S_n=857.5$ $S_n = \frac{n}{2}(a+t_n)$ $857.5 = \frac{n}{2}(7+42)$ $857.5 = \frac{n}{2}(49)$ $35 = n$
c) $d=3.5, \;\; n=13, \;\; S_n=416, \;\; a=?$ $S_n=416, d=3.5, n=13$ $t_8 = 13+(8-1)4$ $t_8 = 13+28$ $t_8 = 41$ $S_8 = \frac{8}{2}(13+41)$ $S_8 = 4(54)$ $S_8 = 216$	d) $d=-7, \;\; n=16, \;\; S_n=-600, \;\; a=?$ $d=-7, S_{16}=-600, n=16$ $S_n = \frac{n}{2}(2a+(n-1)d)$ $S_{16} = \frac{16}{2}(2a+(16-1)(-7))$ $-600 = 8(2a-105)$ $-75 = 2a-105$ $15 = a$
e) $a=12, \;\; S_{15}=549, \;\; d=?$ $a=12, S_{15}=549, n=15$ $S_n = \frac{n}{2}(2a+(n-1)d)$ $S_{15} = \frac{15}{2}(2\times 24+(15-1)d)$ $549 = \frac{15}{2}(48+14d)$ $73.2 = 48+14d$ $1.8 = d$	f) $a=4.8, \;\; n=12, \;\; d=5/3, \;\; S_{12}=?$ $a=4.8, n=12, d=5/3$ $S_n = \frac{n}{2}(2a+(n-1)d)$ $S_{12} = \frac{12}{2}(2(4.8)+(12-1)5/3)$ $S_{12} = 6(9.6+11(5/3))$ $S_{12} = 167.6$

3. Given the equation of the general term, find the indicated sums:

a) $t_n = 5 + 3n$ $\quad S_{12} = ?$ $\quad (Sum\ of\ first\ 12 terms)$

Use the general formula to find t_1 and t_{12}

$t_1 = 5 + 3(1)$
$\quad = 8$
$t_{12} = 5 + 3(12)$
$\quad = 41$

$S_{12} = \frac{n}{2}(t_1 + t_{12})$
$S_{12} = \frac{12}{2}(8 + 41)$
$S_{12} = 294$

b) $t_n = 7 + 8n$ $\quad S_9 = ?$ $\quad (Sum\ of\ first\ 9\ terms)$

Use the general formula to find t_1 and t_9

$t_1 = 7 + 8(1)$
$\quad = 15$
$t_9 = 7 + 8(9)$
$\quad = 79$

$S_9 = \frac{9}{2}(t_1 + t_{12})$
$S_9 = 4.5(15 + 79)$
$S_9 = 423$

c) $t_n = 8 + (n-1)12$ $\quad S_{20} = ?$ $\quad (Sum\ of\ first\ 20 terms)$

By looking at the general term, we can find "a"and "d"

$a = 8, d = 12, n = 20$
$S_n = \frac{n}{2}(2a + (n-1)d)$
$S_{20} = \frac{20}{2}(2 \times 8 + (20-1)12)$
$S_{20} = 10(16 + 228)$
$S_{20} = 2440$

d) $t_n = -\frac{12}{5} + (n-1)\frac{2}{3}$ $\quad S_{25} = ?$ $\quad (Sum\ of\ \text{first}\ 25 terms)$

By looking at the general term, we can find "a"and "d"

$a = -12/5, d = 2/3, n = 25$
$S_n = \frac{n}{2}(2a + (n-1)d)$
$S_{25} = \frac{25}{2}(2 \times (-2.4) + (25-1)2/3)$
$S_{25} = 12.5(-4.8 + 16)$
$S_{25} = 140$

4. The sum of the first 8 terms of an arithmetic sequence is 34 and the sum of the first 9 terms is 38. What is the value of the 9th term?

The 9th term is equal to the first 9 terms minus the first 8 terms

$t_9 = S_9 - S_8$
$t_9 = 38 - 34$
$t_9 = 4$

5. An arithmetic series has 24 terms. The sum of the t_2 and t_{23} is 45. What is the sum of all the terms?

If you pair up the first and last terms, second term and second last term, the sums are equal.

$$S_{24} = \frac{24}{2}(t_2 + t_{23})$$
$$S_{24} = 12(45)$$
$$S_{24} = 540$$

6. Given the two arithmetic series, which one has a greater sum?

$S_1 = 3 + 7 + 11 + ... + 87$ OR $S_2 - 8 + -3 + 2 + 7 + ...97$

First find out how many terms are in each sequence:

$S_1 : a = 3, d = 4, t_n = 87$
$t_n = a + (n-1)d$
$87 = 3 + (n-1)4$
$21 = n - 1$
$22 = n$

$S_1 = \frac{22}{2}(3 + 97)$
$S_1 = 11(100)$
$S_1 = 1100$

$S_2 : a = -8, d = 5, t_n = 97$
$t_n = a + (n-1)d$
$97 = -8 + (n-1)5$
$21 = n - 1$
$22 = n$

$S_2 = \frac{22}{2}(-8 + 97)$
$S_2 = 11(89)$
$S_2 = 979$

7. Rogers charges \$50 a month and \$0.10 a minute. Bell charges \$25 a month and \$0.25 a minute. If Sharon uses 100 minutes a month, which cell phone provider would be cheaper for her?

Find the cost for each cell phone provider if 100 minutes were used each month

$Monthly\ Cost\ at\ Rogers$
$Cost = \$50 + 0.10(100)$
$C = 50 + 10$
$C = \$60$

$Monthly\ Cost\ at\ Bell$
$Cost = \$25 + 0.25(100)$
$C = 25 + 25$
$C = \$50$

The monthly cost at Bell would be cheaper

8. If Sharon uses 5 additional minutes each month, which provider would be cheaper after 3 years? What is the total cellular cost of 3 years for each plan?

There is 36 months in 3 years, so n=36. The cost for each month increases by the same value, so it is an arithmetic series
At Rogers, 5 additional minutes costs $0.50. At Bell, 5 additional minutes costs $2.50.

Total Cost at Rogers

$$TotalCost = \frac{36}{2}\left(2(60)+(36-1)0.50\right)$$
$$TC = 18(120+17.5)$$
$$TC = \$2475$$

Total Cost at Bell

$$TotalCost = \frac{36}{2}\left(2(50)+(36-1)2.50\right)$$
$$TC = 18(100+87.5)$$
$$TC = \$3375$$

9. What is the sum of the first 50 positive odd integers? Write a formula for the first 'n" positive odd integers:

The first odd integer is 1, so $t_1 = 1$. Each consecutive odd integer goes up by 2, so $d = 2$.

$$S_{50} = \frac{50}{2}\left(2(1)+(50-1)2\right)$$
$$S_{50} = 25(2+98)$$
$$S_{50} = 2500$$

10. What is the sum of the first 100 positive even integers? Write a formula for the first "n" positive even integers

The first even integer is 2, so $t_1 = 2$. Each consecutive even integer goes up by 2, so $d = 2$.

$$S_{100} = \frac{100}{2}\left(2(2)+(100-1)2\right)$$
$$S_{100} = 50(4+198)$$
$$S_{100} = 10100$$

11. What is the sum of all the multiples of 7 between 10 and 200?

The first multiple of 7 greater than 10 is 14 or $7(2)$. The biggest multiple of 7 less than 200 is 196 or $7(28)$

So the sum of all multiples of 7 from 10 to 200 will be: $7(2)+7(3)+7(4)+7(5)+.....7(19)+7(20)$

If we look at the series this way, then $t_1 = 14$, $d = 7$, $n = 20-2+1 = 19 terms$

$$S_{19} = \frac{19}{2}\left(2(14)+(19-1)7\right)$$
$$S_{19} = 9.5(28+126)$$
$$S_{19} = 1463$$

12. What is the sum of all the multiples of 12 between 100 and 1000?

This question is the same as the previous one. The answer is:

$$S_{75} = \frac{75}{2}\left(2(108)+(75-1)12\right)$$
$$S_{75} = 37.5(216+888)$$
$$S_{75} = 41400$$

13. The sum of the arithmetic series (-300)+(-297)+(-294)+…+306+309 is

a) 309 b) 927 c) 615 d) 918 e) 18

The series looks like this: $-300+(-297)+(-294)+.....+(294)+(297)+(300)+(303)+(306)+(309)$

Many of the terms cancels out (positive and negative)

So only three terms remain: 303, 306, and 309. The sum will be 918

14. Four numbers are in an arithmetic sequence and their sum is 82. The third term is three times bigger than the first term. How much is the 4th term greater than the 2nd term?

If $t_3 = 3 \times t_1$

Then

$$a + 2d = 3a$$
$$2d = 2a$$
$$d = a$$

$$t_1 + t_2 + t_3 + t_4 = 82$$
$$(a) + (a+d) + (a+2d) + (a+3d) = 82$$
$$4a + 6d = 82$$
$$10d = 82$$
$$d = 8.2$$

To go from t_2 *to* t_4, increase by 2 common difference

$$t_4 - t_2 = 2d$$
$$= 16.4$$

15. The first term of an arithmetic series is 6, the common difference is 7, and the sum is 2814. How many terms are in the series?

$$2814 = \frac{n}{2}(2(6) + (n-1)7)$$
$$2814 = \frac{n}{2}(12 + 7n - 7)$$
$$5628 = n(5 + 7n)$$

Since 'n" must be an integer, you can guess and check. If you know how to use the quadratic formula, you can use that also. n=28

16. The first three terms of an arithmetic sequence is given by the following expressions: $2x+1,\ 4x,\ 5x+2$. Find the sum of the first 10 terms

$$t_2 - t_1 = t_3 - t_2$$
$$4x - (2x+1) = 5x + 2 - 4x$$
$$2x - 1 = x + 2$$
$$x = 3$$

$$t_1 = 7$$
$$t_2 = 12$$
$$t_3 = 17$$
$$a = 7, d = 5, n = 10$$

$$S_{10} = \frac{10}{2}(2(7) + (10-1)5)$$
$$= 5(14 + 45)$$
$$= 295$$

17. In an arithmetic series, the sum of $a_1 + a_2 = 25.5$ and $a_3 + a_4 = 39.5$. What is the sum of the first 10 terms?

$$a_1 = a$$
$$a_2 = a + d$$
$$a_3 = a + 2d$$
$$a_4 = a + 3d$$

$$(a_3 + a_4) - (a_1 + a_2) = 39.5 - 25.5$$
$$2a + 5d - (2a + d) = 14$$
$$4d = 14$$
$$d = 3.5$$

$$2a + d = 25.5$$
$$2a + 3.5 = 25.5$$
$$2a = 22$$
$$a = 11$$

$$S_{10} = \frac{10}{2}(2(11) + (10-1)3.5)$$
$$= 5(22 + 31.5)$$
$$= 267.5$$

